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Modern requirements on constructions impose that proper design strategies must be adopted in order to obtain a robust structure: in this sense, consequence-based design focuses the attention on the structural response to damage. The behavior of statically indeterminate structural systems under damage is nonlinear because the load paths intertwine each other, even if each component behaves linearly. The paper aims both to highlight the behavior of a metallic truss under progressive damage and to define a possible strategy for designing a truss that is able to sustain damage acting at random on one of its elements. Structural complexity is used as a leading parameter. Following the results of a parametric analysis, it emerges that, as much as the Normalized Structural Complexity Index increases, the efficacy of the load paths is spread such that the impact of random damage decreases, making the approach feasible.

1 Introduction

The modern requirements in structural design impose that a structure has to be robust. Many definition of structural robustness have been formulated. ISO (1998) considers the possibility of a structure not to be damaged to an extent disproportionate to the original cause. The Eurocode proposes a similar idea, considering the ability of the structure to withstand events, instead of being damaged (CEN, 2006). The American General Services Administration proposal relates to the concept of the resistance to damage without premature and/or brittle failure (ARA, 2003). The Joint Committee on Structural Safety's document proposes an approach based on risk at a damage state (JCSS, 2011). Many authors have dealt with the concept of robustness proposing various properties that define a structure as robust (Starossek and Haberland, 2011; Agarwal and England, 2008; Biondini et al., 2008a; Bontempi et al., 2007; Val et al., 2006). Vrouwenvelder (2008) states that a robust structure should not be too sensitive to local damage, whatever the source of damage.

In the majority of the ideas previously reported, the concept of damage represents the central idea, i.e., it plays a fundamental role. The actual design approach considers, first, the set of external forces acting on a structure and combines their effects in order to get a spectrum of actions on each element. The structural safety is thus assessed through a reliability-based approach. The preceding approach is not adequate for considering the possibility of progressive collapse, i.e., accounting for robustness. Starossek and Wolff (2005) criticize the assumption that low probability events and unforeseeable incidents (accidental circumstances) need not be taken into consideration in the design, while they are the most dangerous for the construction.

The philosophy to be followed in the design of a structure robust against damages differs from what is usually done. The idea of implementing a design based on the consequences rather than on reliability takes its origins at the beginning of the new millennium. In a conference held at the University of Notre-Dame, IN, Abrams et al. (2002) coined a new term: consequence-based engineering. Despite the fact that they relate the idea to seismic risk, the approach was outlined in its essential aspects. The so-called consequence-based design is composed by an iterative assessment of the consequences of a damage: if anticipated consequences exceed tolerable ones, redesign is necessary until the trend is opposite. The consequences can be estimated for a number of different system-intervention strategies with various input parameters describing the hazard or the built environment (Abrams, 2002).

The concept of damage is, in itself, non-trivial. It can be considered as an unplanned variation of the properties (Mises, 1923) or of the geometry of one or more parts of a structure that entails a weakening and, usually, negative consequences. The methods usually used in the evaluation of damage on a structure consider its static or dynamic response (Andreaus et al., 2007; Andreaus and Baragatti, 2009; Roveri and Carcaterra, 2012), or both (Irschik, 2002). Yao et al. (1986) underlined the fact that the causes of damage can be various: material, structural configuration and construction, loading conditions. Environmental conditions might play a relevant role as well (De Biagi and Chiaia, 2013a; Cennamo et al., 2014). The previous observations can be related to any kind of civil structure. Steel structures, such as reticular masts, bridges, and long-span beams, are prone to be subjected to damages with the possibility of local and global collapses Biondini et al. (2008b). In a society in which anthropogenic hazards are possible, in recent times, specific attention has been paid to the response of structures to unexpected events, e.g., terroristic attacks. These scenarios are unforecastable and the basic hypotheses of reliability-based design are false since the probability of occurrence of the cause of damage is not known *a priori*.

The present paper addresses two important issues. The first relates to the behavior of a truss under progressive damage. In the framework of structural robustness, the relationship between the damage and the structural response would increase the possibility to assess the presence of damage in the structure as long as the damage phenomenon acts on it. The second investigates the possibility to design a truss structure that is able to sustain damage acting at random on one of its elements.

In the present paper, the concept of structural complexity is used. Although a general treatment on the topic is available in De Biagi (2014), a short theoretical reminder is presented in Section 2. Simulations on a sample truss cantilever are illustrated in order to respond to the addressed questions: Sections 3 and 4 detail the calculations and the results, respectively. The results are discussed in Section 5. The approach herein proposed might be implemented in the preliminary steps of a design process. It explores the possibility of increasing the damage-tolerance of the structure by optimizing the variety of load paths under a specific loading scenario.

2 Theoretical background

Two parameters are used throughout the paper. The first accounts for the distribution and the efficacy of the load paths in the structure. The second is related to the behavior of a damaged structure, as detailed in the following.

2.1 Structural complexity

The use of graph theory in the field of structural engineering dates back to the Fifties. The first applications of topology and graph theory to structural mechanics are due to Carter (1944) and Kron (1962), who first made an explicit analogy between electrical networks and elastic structures. In the same period, Langefors (1950, 1956a,b) presented a framework for the analysis of statically indeterminate continuous frames by means of algebraic graph theory. An alternative approach was proposed by Samuelsson (1962) for skeletal structures, and Wiberg (1970) for continuum problems. Henderson and Bickley (1955) related the degree of static indeterminacy of a rigid-jointed frame to the First Betty Number and Kaveh (1988) applied many graph theoretical concepts to structural mechanic and, in particular, to structural optimisation (Kaveh, 2004). Others applications of graph theory to elastic systems can be found in Kaveh (2006).

In mathematics, Rashevsky (1955) introduced the notion of topological content, which was formalized by Mowshowitz (1968) through the concept of graph entropy. Following that, the interest for information-theoretic network complexity increased and the concept of graph entropy was applied to many disciplines (Dehmer and Mowshowitz, 2011; Mowshowitz and Dehmer, 2012). In recent times, a generalized framework for network complexity was proposed (Dehmer and Mowshowitz, 2010).

Recently, De Biagi and Chiaia (2013a) defined a complex structure as a system made up of a large number of parts that interact in a non-simple way under an arbitrary loading scheme. This definition, following the work by Simon (1962) in other disciplines, accounts for the shape of the structure, its stiffness, and the acting loads. The metrics for determining the structural complexity are based on the so-called Information Content introduced by Shannon (1948) and implemented in the researches on graph entropy previously recalled. Here, the information content is represented by the effectiveness of the load paths across the truss. A simple structure is the one that has a reduced number of effective load paths. On the contrary, when all the possible load paths are equally effective, the structure reaches its maximum complexity (De Biagi, 2014).

A path for the loads between the elevation nodes and the foundation ones is conceptually materialized as a fundamental structure, i.e., a link between the elevation and the foundation. The load path, and thus a fundamental structure, is determined through the law of statics. In truss structures, a fundamental structure is a statically determinate scheme of rods that spans all the nodes and is made of a subset of rods of the reference truss. In frame structures, the fundamental structures were originated from cuts turning the frame into a tree-like structure. Herein, the extraction of fundamental structures from a statically indeterminate truss is performed through the alternate removal of rods.

The elastic energy, or the deformation work, is the parameter that better describes the behavior of a structure subjected to loads. First, it accounts both for stiffness and loads, and, in case of nonlinear analysis, it considers the ductility of the elements composing the structure. The effectiveness of a load path, identified through the fundamental structure, is measured as the ratio between the deformation work in the reference structure and the one performed on the fundamental structure. This ratio is called the performance ratio ψ and ranges from 0 to 1 since the denominator is always larger than the numerator. If the deformation work of the fundamental structure is close to the one of the reference, the load path results effective; if the deformation work of the fundamental structure is significantly larger than that of the reference scheme, the ratio tends to zero, meaning that the load is not effective, i.e., not representative of the overall behavior of the statically indeterminate structure. The number of fundamental structures and, consequently, of performance ratios, n, depends on the original scheme. The measure of the “amount” of information required to describe the structural behavior, is based on the definition of information entropy stated by Shannon (1948). In particular, the Structural Complexity Index SCI, is represented by

$$\mathrm{SCI} = -\sum_{i=1}^{n} \left(\frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \log \frac{\psi_i}{\sum_{j=1}^{n} \psi_j} \right), \tag{1}$$

where ψ_i is the performance ratio of the i-th fundamental structure, as defined previously. The base of the logarithm is not relevant (if 2, the measure is in bit). The entropy measure possesses many interesting properties (Gray, 2011). The identification of the load paths can be easily performed if the structural scheme is studied under the framework of Graph Theory (De Biagi and Chiaia, 2013a).

In order to compare the complexities of various structures with different sizes and element numbers, a normalized parameter is introduced. The SCI is divided by its maximum possible value, which represents the situation in which each possible load path has the same effectiveness (i.e. the same performance factor). This situation, representing the maximum complexity, corresponds to a SCI equal to $\log n$, where n is the number of fundamental structures (load paths). Thus, the Normalized Structural Complexity Index, NSCI, is expressed as

$$\mathrm{NSCI} = \frac{\mathrm{SCI}}{\log n}. \tag{2}$$

The NSCI ranges between 0 and 1. As much as the parameter approaches to 0^+, the structural system is simple. On the opposite side, values of NSCI tending to 1^- refer to complex structures (De Biagi and Chiaia, 2013a).

Structural complexity presents scaling invariance properties under specific loading cases, i.e., presence of forces, exclusively, or couples, exclusively (De Biagi and Chiaia, 2013b, 2014). In linear elastic truss structures that are supposed to be loaded by forces acting on the nodes, a geometric scaling (on lengths, sizes, and areas) does not vary the value of the structural complexity. An identical result is obtained by scaling the magnitude of the external loads.

The Normalized Structural Complexity Index does not deal with damage, but it is closely related to. In fact, in comparing a structure before and after the removal of one of its members, the fundamental structures of the damaged structure are a subset of the fundamental structures of the undamaged one. In De Biagi and Chiaia (2015) is detailed that structures made by rods in parallel behave in a completely different manner depending on the way the axial stiffnesses are assigned to each member. On the opposite side, in structures made by elements arranged in series, the removal of an element causes the global failure. Frames and trusses are, in a certain sense, in an intermediate configuration between the two previous ones. Despite the way the elements are arranged in the structure (i.e., the topology), the response of the structure to damage is closely related to the way the stiffness are distributed among the members. In this sense, the effects of a random damage are quite different.

2.2 Progressive damage and M-value

It is possible to introduce damage in a structural scheme in various ways. In a linear elastic structure, the reduction of the Young's modulus through a damage parameter, as proposed by Lemaître and Chaboche (1994), implies a proportional reduction of the axial stiffness of the damaged element. In parallel, the same effect can be obtained through the reduction of the cross-section area of the rod, as suggested by Biondini et al. (2008b). This approach is easily applicable to trusses, but may unsuitable for frame structures and the appropriate description of the damage pattern must be done after the definition of physical degradation process.

Herein, in order to assess the impact on the structural system of a progressive damage on one of its components, the value of the axial stiffness was progressively reduced through a damage parameter ξ ranging from 0 (no damage) to 1 (element removal). The axial stiffness of the i-th damaged element is computed as

$$K_{i,\xi} = K_0 \, (1 - \xi) \tag{3}$$

A much as the damage acts on the structure, the elastic energy tends to increase (De Biagi, 2014; De Biagi and Chiaia, 2015). Elastic energy, i.e., deformation work, is used as a measure of the behavior of the structure under damage. Between all the possible metrics for assessing the robustness (Biondini and Restelli, 2008; Starossek and Haberland, 2011), the elastic energy presents many positive properties: (i) deformation work is not affected by load history but only by the initial and final positions since pure elastic structures (linear and non-linear) are conservative systems. (ii) Its computation is simple: in linear elastic structures it is equal to the work performed by external forces and can be computed by Clapeyron's Theorem. (iii) The elastic energy merges into a single quantity: the stiffness of the structure and the loads acting on it (De Biagi and Chiaia, 2013a). Considering that the external loads do not change throughout the damage process, (iv) a variation in deformation work implies a variation in displacements. Its variation can be intended as a robustness measure.

Various approaches for measuring the robustness have been proposed. Starossek and Haberland (Starossek and Haberland, 2008, 2011) introduced an energy-based metric based on the evaluation of the energy required for damage propagation. In parallel, they suggested a stiffness-based measure that accounts for the ratio between the determinant of the stiffness matrix of the structure deprived of the damaged element (or connection) and the determinant of stiffness matrix of the intact structure. Biondini and Restelli (2008) proposed and compared the effectiveness of various structural performance indicators: they found that the ratios between either displacements or stored energies in the undamaged and damaged configurations are suitable for damage-tolerance analysis.

In the present analysis, in order to keep track of the variation of the deformation work as much as the damage parameter ξ varies, the increment of elastic energy when the i-th rod is subjected to progressive damage, Ω_i, is computed as

$$\Omega_i = \frac{W_{i,\xi} - W_0}{W_0}, \tag{4}$$

where W_0 is the deformation work in the reference structure, while $W_{i,\xi}$ is the deformation work when the i-th rod is subjected to a damage level ξ. The value is dimensionless since it is a ratio of energies.

The average structural response in terms of deformation work is given by the value M. This parameter quantifies the average effect of the removal of a single arbitrary rod of the truss and is computed as

$$\mathrm{M} = \frac{1}{r} \sum_{i=1}^{r} \frac{W_{i,1} - W_0}{W_0}, \tag{5}$$

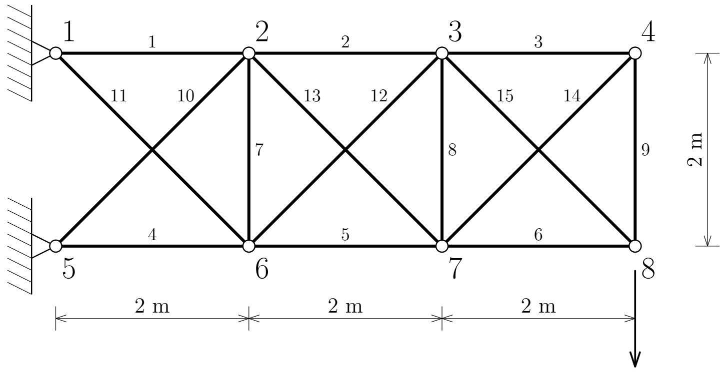


Figure 1: Schematic of the truss used in the analysis. The nodes and the rods are numbered.

where r is the number of rods. Note that at the quantity in the numerator represents the increment of deformation work when the i-th rod is removed (De Biagi and Chiaia, 2015).

3 Methods

The previous concepts have been applied for the analysis of a truss system. All the calculations were done in Matlab through specific scripts. The implemented model consider that: the material is linear elastic and that the cross-section area does not changes along the length of the elements. The weight of the rod is neglected. The external actions are represented by nodal loads, exclusively.

The tests were made on a statically indeterminate truss structure, see Figure 1. The scheme is composed by 15 rods; the external supports are represented by two pins at nodes no. 1 and 5; the joints, i.e. internal hinges, are six. The nodes span a grid of 2 m spacing. In the reference structure, the material Young's modulus is set at 210 GPa, the reference cross-section is set at $9.654 \times 10^{-4}\,\mathrm{m}^2$, corresponding to 88.9 x 3.6, EN 10210 structural hollow tubular section. The scheme is loaded by a downwards vertical force (of magnitude equal to 50 kN) acting at node no. 8. By virtue of scaling properties, the choice of the load is arbitrary, as well the size of the truss.

The structure of Figure 1 is a good compromise between a simple structure, for which the engineer acquainted with structures can grasp the behavior, and a redundant structure. The choice of making the analysis on trusses rather than on frames relates to the number of forces within the rods (just axial force) rather than in the beams (axial and shear forces and bending moment for plane structures).

Damage on the structure was introduced through the damage model represented by Eqn. (3), as described in Section 2. The resultant structure was solved in order to measure nodal displacements and axial forces in the elements.

The structural complexity (NSCI parameter) was determined through the computation of the deformation works of both the undamaged scheme and the fundamental structures. The extraction process consists in making a partial copy of the reference structure that is only made of those elements belonging to the fundamental structure. Thanks to the numerical approach based on rods (rather than on beams), the assessment of the internal stability (presence of unsupported rotational degrees of freedom) is unnecessary. The M-value was determined through the alterative removal of one element in the reference structure and the computation of the corresponding deformation work.

A parametric analysis was performed on the truss scheme previously illustrated in order to asses the presence of a relationship between the Normalized Structural Complexity and the M-value. The procedure consists in generating a set of cross-section areas and evaluating in a structural scheme made of such elements the previous parameters, i.e., NSCI and M. Because of the scaling properties of structural complexity, the stiffness of one element needs to be kept fixed throughout the analysis (De Biagi, 2014). The cross-section area of rod #1 was set equal to $1 \times 10^{-3}\,\mathrm{m}^2$. A uniform random generator was used. The upper bound of each generated value was fixed at $2 \times 10^{-2}\,\mathrm{m}^2$. The external load on node no. 8 was kept constant at 50 kN throughout the whole analysis.

4 Results

4.1 Progressive damage

The truss of Figure 1 was subjected to progressive damage acting on Young's modulus through parameter ξ. The increase of damage parameter ξ implies an increase of elastic energy, i.e. the deformation work. Figure 2(a) plots the increment of deformation work, Ω, with respect to the damage parameter.

As shown in Figure 2(a), the progressive damage, measured through parameter Ω, is non linear. Different behaviors are present. For rods no. 1 and 4 the major increment of deformation work occurs for ξ larger than 0.70: from $\xi = 0$ to 0.70 only roughly 20% of the total increment is recorded. A similar trend is seen for rods no. 2 and 5: the slope of the graph increases progressively. The damage of the remaining rods produces smaller increments in deformation work: the maximum observed value of Ω is 1.135 (rods no. 6 and 15). A detail is reported in the box of Figure 2(a). The damage on rods no. 6 and 15 leads to an equal value of Ω at $\xi = 1$, while the trends are different for smaller damage parameters (the slope of the curve of Ω_{15} is not affected by the damage parameter). A similar trend is observed for rods no. 3, 9 and 14. The damage on rods no. 10 to 13 results in a progressive increment of the deformation work. Damage on rods no. 7 and 8 does not imply any increment in deformation work.

The average of the increments of deformation work, i.e., the M-value, is equal to 0.5158. The number of fundamental structures is equal to 80. The Structural Complexity Index (SCI) is equal to 1.8649, while the Normalized Structural Complexity Index (NSCI) is equal to 0.9771.

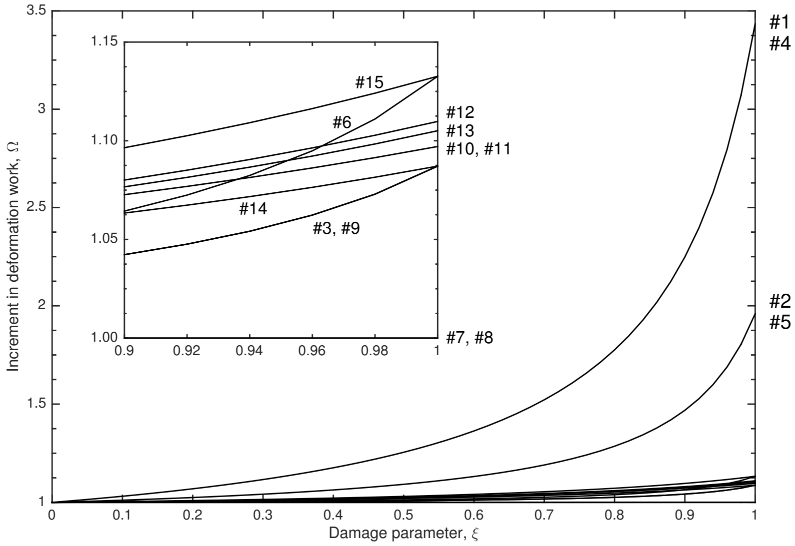
The axial forces in the elements were assessed in evaluating the effects of damage on one element. Figure 2(b) shows the global maximum and minimum axial forces in the rod after the damage process. Tension forces are positive. Independently from the damage, rods no. 1 to 3 are always in tension. The largest axial force ranges are observed as much as the rod is closed to the external support. Same results (with negative sign, i.e., compression) are obtained for rods no. 4 to 6. Rods no. 7 to 9, the vertical elements, can be alternatively in tension or in compression. The axial force in the oblique rods (no. 10 to 15) changes sign, with increasing variability as much as the considered element approaches the support. Rod no. 1 has the maximum tensile force (250 kN); rod no. 4 has the maximum compression force (−250 kN). The largest variability belongs to rods no. 10 and 11 (the range width is about 176.8 kN).

4.2 Parametric analysis

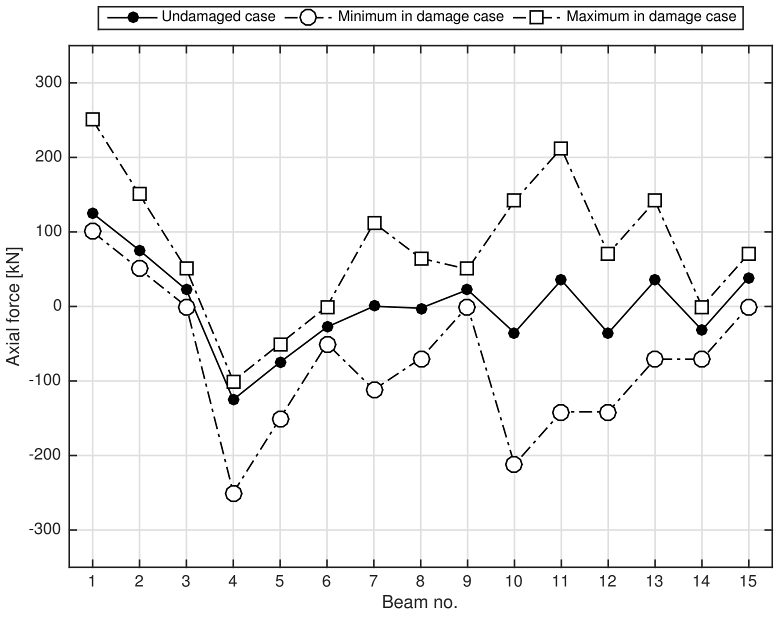
The parametric analysis was done on a sample of 2×10^5 structures. The recorded values of NSCI range between 0.7891 and 0.9993; M-value ranges between 0.1812 and $6.715 \times 10^3 \cdot$ Each point in Figure 3 is representative of a generated structural scheme, onto which the complexity was computed and the M-value determined. In order to catch the behavior for small values of M, the ordinates are in a logarithmic scale. The cloud of points has considerable variations in density: 75% of the points are in the range of NSCI $= (0.93; 0.98)$ and M $= (0.3; 1.5)$.

Different trends are recorded, see Figure 3: for the sake of simplicity, the limit cases are denoted with capital letters A to D. Limit cases A and B are represented by an asymptotic increase of the M value. Limit case C is related to a truncated distribution for relatively low values of M; limit case D is related to the rightmost part of the distribution.

Limit cases A and B Limit cases A and B are represented by asymptotic tails in the distribution. A detailed trend is shown in Figures 4(a) and 4(b). Referring to case A, a single tail is present. A vertical asymptote is supposed at a value of NSCI equal to 0.785. Referring to case B, seven tails are highlighted (namely B_1 to B_7). Table 1 reports the cross-section areas (dimensionless values) related to the trusses computed in the parametric analysis in cases A and B. The reported values are the ones related with the the highest recorded value of parameter M, for each case. The corresponding values of NSCI and M are reported at the bottom of Table 1. Zero values of cross-section areas are observed throughout the table. In case A, a quasi-null cross-section area on rod no. 14 presupposes that any damage acting on rods no. 15 or 6 implies large vertical displacements of the loaded node (node no. 8 in Figure 1). In case B_1, where rod no. 10 has axial stiffness close to zero, damage on rod no. 4 engenders large displacements since all the cantilever (supposed as a rigid body) rotates around the top



(a) Increment of deformation work for progressive damage



(b) Axial forces in the elements pre- and post- damage

Figure 2: Effects of damage on the reference truss of Figure 1. The box of Figure 2(a) is a detail of the rightmost part of the diagram for values of M close to one.

pin support (node no. 1). A similar situation occurs in case B_2 for which rod no. 11 has cross-section area close to zero. Here, for example, progressive damage on rod no. 1 presupposes large displacements up to the formation of a mechanism around node no. 5. Limit situation B_3 relates to similar collapse mechanisms for a limited part of the truss cantilever. In this case, where rod no. 2 has quasi-zero axial stiffness, either progressive damages on inclined rod no. 12 or on rod no. 13 presuppose large displacements and the quasi-unstable structure (the reference to quasi-unstable is due to the fact that, during the damage, the rod with negligible cross-section area is still present in the structural scheme, which results in any case statically determinate). Damage on rod no. 5 does not imply the formation of a potential mechanism. Similar limit analysis can be done for the remaining structures belonging to limit situation B. Tails B_4 and B_5 are very close to each others.

Limit case C Limit case C, illustrated in detail in Figure 4(c), is represented by a cut in the cloud of solutions. The estimated position of the vertical cut is at NSCI ≈ 0.864. No points are found at lower complexities for values of M smaller than 3.5 (this last point is part of the tail of case A). In order to highlight the distribution of stiffnesses of the trusses belonging to this case, a statistical representation is proposed. Figure 5(a) shows a box plot of the dimensionless cross-section sizes of the rods. Rod no. 1 has unit cross-section area. The statistically analyzed trusses are the ones for which NSCI < 0.876 and M < 1.50. In the parametric analysis performed, the number of cases within the previous bounds is 136. In the box plot, the median (namely, q_{50}) is in red. The bounds of the box are at the 25% and 75% percentiles, i.e., q_{25} and q_{75}, respectively. The upper whisker is at $q_{75} + 1.50\,(q_{75} - q_{25})$, the lower at $q_{25} - 1.50\,(q_{75} - q_{25})$. If the data are normally distributed, the bounds of the whiskers encompass the 99.3 % of the cross-section sizes. The boxes are notched. The lower and upper extreme of the notches are at $q_{50} \pm 1.57\,(q_{75} - q_{25})\,/\sqrt{136}$, respectively (136 is the cardinality of the dataset). Two medians are significantly different at the 5% significance level if the respective notches do not overlap (McGill et al., 1978). The crosses relate to the outlier data. The most relevant result from Figure 5(a) is that the cross-section size of rod no. 7 is null.

Limit case D Limit case D, illustrated in detail in Figure 4(d), is referring to the rightmost part of the distribution, where the complexities are close to 1. For values of NSCI larger than 0.99, the greatest observed value of M is bounded at 0.6. The minimum value of M is at 0.1812. For lower values of complexity, some truss structures exhibit low values of M close to 0.2. Following the same approach as in case C, a subset of data of size of 33, with NSCI > 0.99 and M < 0.26, was analyzed statistically. Figure 5(b) shows a box plot of the dimensionless cross-section sizes of the rods. In this situation, none of the medians has null value. In general, the cross-section sizes of vertical and diagonal rods are larger than the upper and lower chords.

A truss structure in which rod cross-section areas are the medians obtained in the previous analysis was tested. The normalized area of the rods is reported in Table 1. Figure 6(a) shows the increments of the deformation work for progressive damage on the truss structure previously detailed. The major increments of deformation work occurs for damage on rods no. 1 and 4, with Ω equal to 1.8. The nonlinearity of the response is still present, but it is attenuated with respect to the situation reported in Figure 2(a). At $\xi = 0.70$, 40% of the total increment is recorded. A similar result is obtained for damage on rods no. 2 and 5. Damage on the other rods results in a progressive increase of deformation work with a maximum increment equal to $\Omega = 1.063$ for rod no. 13. The axial forces on the rods in the undamaged structure and the global maximum and minimum after the damage process are shown in Figure 6(b). Results similar to the ones of Figure 2(b) are obtained.

5 Discussion

The first analysis shows that the behavior under progressive damage is nonlinear. The rods constituting the reference truss are made of linear elastic material with no yielding. The damage acts linearly on the elastic modulus. Since the structure is statically indeterminate, the redistribution of loads is possible when a variation in mechanical properties occurs. The amount of redistribution depends on the distribution of stiffnesses, which varies as much as the damage progresses. The source of the nonlinearity lies in the previous considerations.

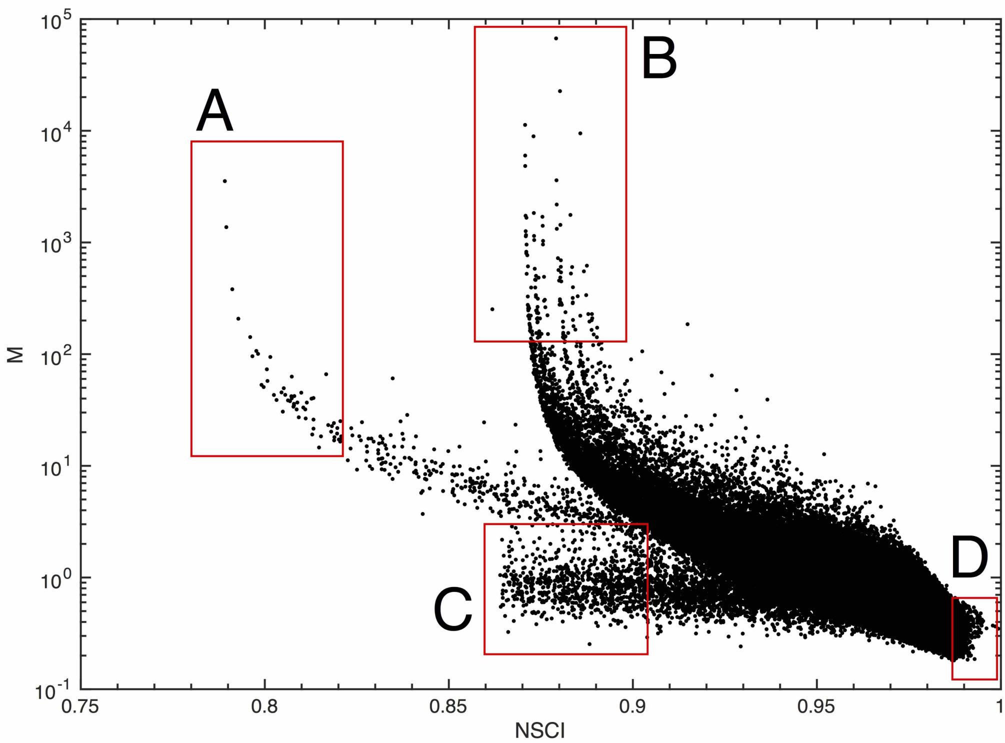
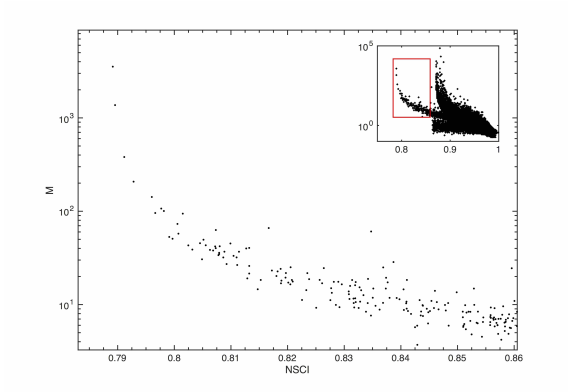
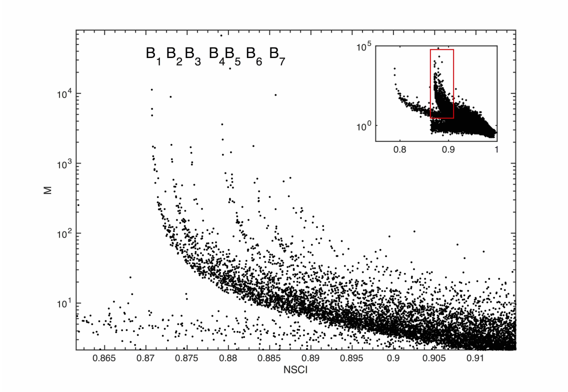


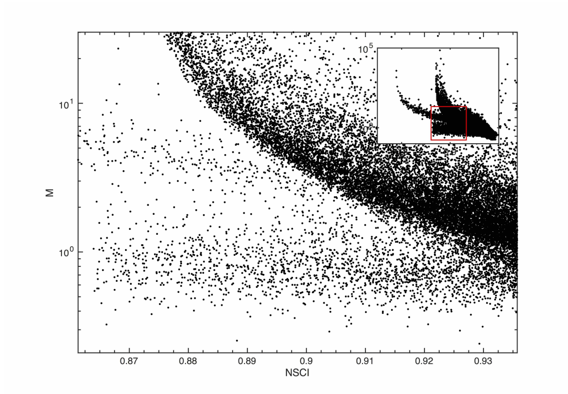
Figure 3: Cloud of dots representing the results of the parametric analysis.



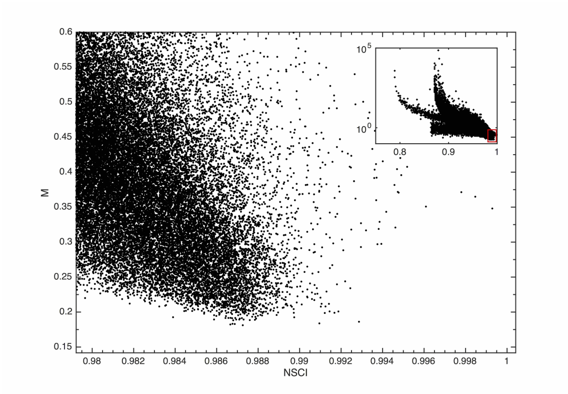
(a) Limit case A



(b) Limit cases B

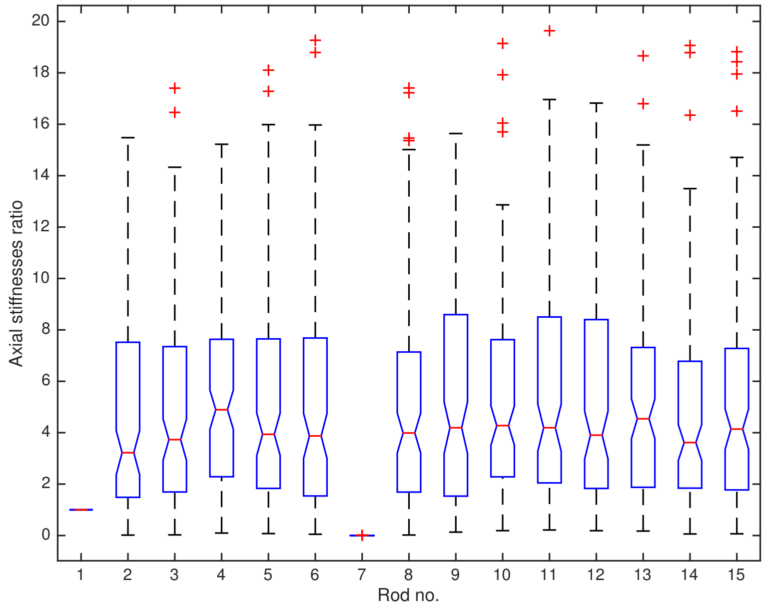


(c) Limit case C

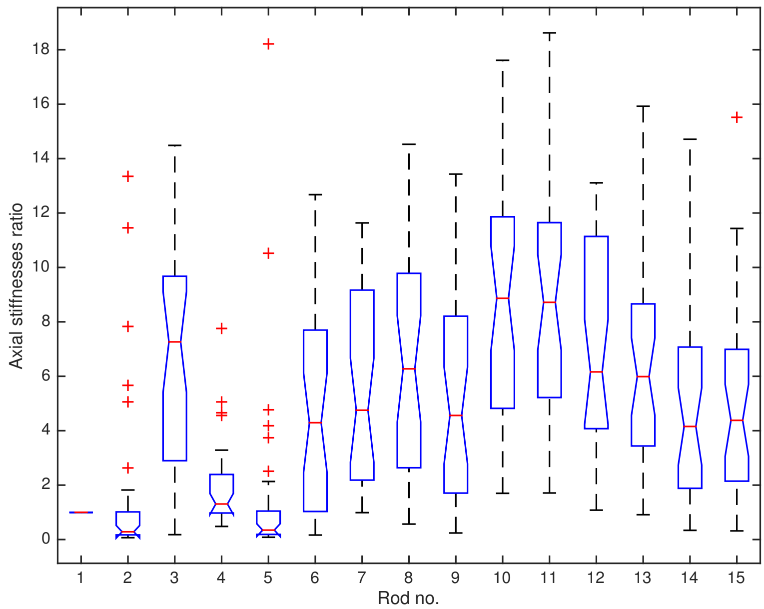


(d) Limit case D

Figure 4: Details of Figure 3.



(a) Limit case C

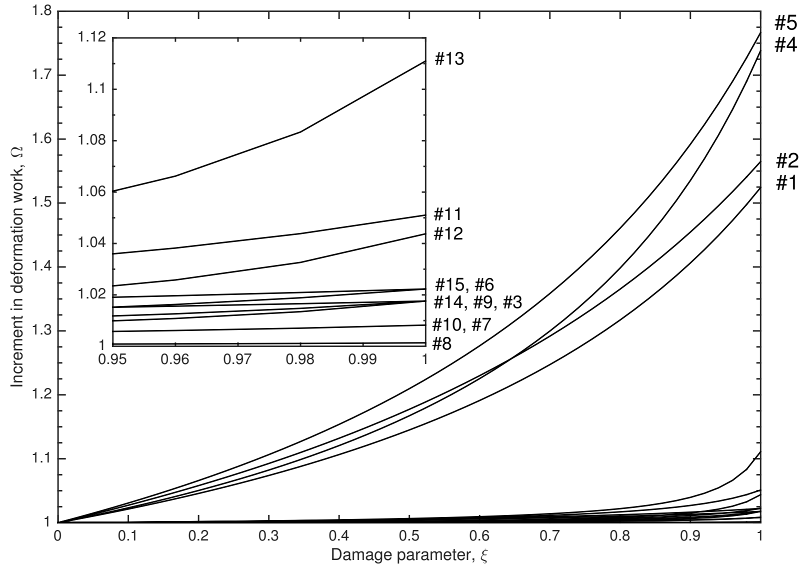


(b) Limit case D

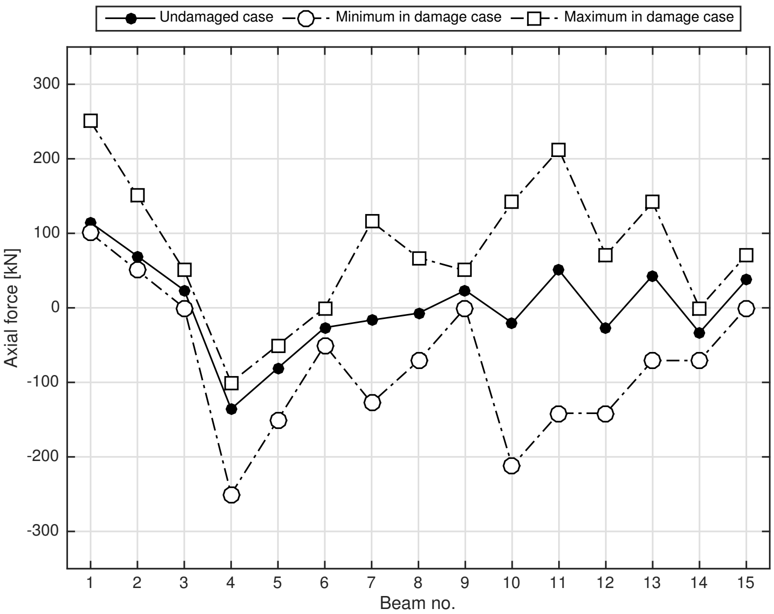
Figure 5: Box plots of the dimensionless cross-section sizes of a subset of data of limit cases C and D. Details on the bounds are in the text.

Rod	Limit case								
	A	B_1	B_2	B_3	B_4	B_5	B_6	B_7	D
#1	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
#2	1.26	0.99	2.69	$\sim$0.00	1.93	5.21	5.87	10.07	0.28
#3	0.88	0.39	0.26	4.74	2.27	3.15	7.98	2.09	7.26
#4	3.15	9.22	1.27	1.41	1.60	$\sim$0.00	18.70	8.07	1.31
#5	2.14	1.47	0.71	6.48	1.88	7.82	16.86	4.57	0.35
#6	2.24	0.39	2.85	3.29	0.43	4.99	4.04	11.44	4.29
#7	0.83	6.31	2.57	15.08	1.82	5.99	13.68	3.36	4.75
#8	2.00	7.72	2.48	9.36	1.37	1.03	13.03	6.74	6.27
#9	3.76	5.47	2.17	9.82	0.88	3.07	2.97	2.28	4.56
#10	3.89	$\sim$0.00	2.04	15.18	0.95	8.73	14.23	1.19	8.87
#11	0.37	0.61	$\sim$0.00	11.32	2.20	5.75	17.47	12.25	8.72
#12	3.35	8.09	1.17	2.73	2.48	7.52	$\sim$0.00	7.78	6.16
#13	3.64	6.92	2.27	14.10	$\sim$0.00	4.21	2.12	2.97	5.99
#14	$\sim$0.00	5.79	0.29	8.03	0.41	4.33	17.04	1.66	4.16
#15	3.25	0.40	2.88	2.93	1.11	4.21	0.70	$\sim$0.00	4.38
NSCI	0.7891	0.8707	0.8730	0.8754	0.8791	0.8802	0.8830	0.8857	0.9924
M	3.5×10^3	1.1×10^4	8.9×10^3	1.7×10^3	6.7×10^4	2.3×10^4	1.8×10^3	9.5×10^3	0.194

Table 1: Cross-section areas related to the truss structures belonging to the limit cases A, and B_1 to B_7, found in the parametric analysis. The sizes related to limit case D are the medians of the simulations for which NSCI > 0.99 and M < 0.26. Rod no. 1 has unit cross-section area: the values are dimensionless through a factor $0.001\,\mathrm{m}^2$. Normalized Structural Complexity and M-value are reported in the bottom.



(a) Increment of deformation work for progressive damage



(b) Axial forces in the elements pre and post damage

Figure 6: Effects of damage on the reference truss which rod cross-section areas are te median values of limit case D, reported in column D of Table 1. The box of Figure 6(a) is a detail of the rightmost part of the diagram for values of M close to one.

The measure of the increment of displacement of the loaded nodes as a parameter for assessing the presence of a progressive damage in the truss leads to incorrect estimates. The effects of the progressive damage on the reference truss, herein graphed as increments of elastic energy Ω, which is, in turn, related to the displacements, are different depending on the damaged element. As shown in Figure 2(a), the damage acting on the main chords produces larger displacements than the damage acting on the oblique and vertical rods. Despite the presence of displacements, the largest proportional increments occur when the element is close to being totally damaged. The last consideration can be clearly noted in the curves referring to rods no. 1 and 4.

In the truss structure proposed, no energy increments are shown for progressive damage acting on rods no. 7 and 8. The axial force in these elements in the reference structure is null; that is why their removal has no overall impact.

Looking at the parametric analysis, the trend that emerges is that the value of parameter M reduces as much as the Normalized Structural Complexity tends to one. In other words, since the NSCI indirectly evaluates the uniformity in the efficiency of the load paths across the structural scheme, an increase of complexity implies a better redistribution under damage. The highlighted asymptotic behaviors reflect the fact that, in the generation of the structures, there is the possibility to assign cross-section areas close to zero to a rod. This fact does not affect significantly the value of the structural complexity since there is still static indeterminacy in the truss. Regardless the damage on a rod belonging to the neighborhood of the rod with low axial stiffness may imply an increase of the deformation work since the displacements increase sharply. In a limit case, i.e., the cross-section area is null, the truss structure turns into a mechanism for $\xi = 1$. Such structure is not robust.

Cross-section areas close to zero have been found on eight rods (Table 1). Theoretically, it is possible to assign quasi-zero cross-section area on rods different from the ones discussed in the previous paragraph (except on rod no. 7, as explained in the following). In such cases, damages on one of the remaining elements presupposes large displacements and, thus, large deformation work. Such cases seem not to be present in the parametric analysis. This is due to the relatively small number of simulations performed. As can be seen in Figure 4(b), there are points on the right-hand side of tail B_7. These can be related to the configurations previously discussed. A large number of simulations would highlight such trends.

The unique exception of the previous considerations relates to rod no. 7. Limit case C, plotted in Figure 4(c), clearly shows that there is the possibility to reduce the complexity of the truss, i.e., reducing the efficacy of the load paths, without necessarily presupposing an increase of M. The box plot of Figure 5(a) illustrates that, for all the 136 analyzed structures belonging to the tail (the bounds are reported in the previous section), the cross-section area of rod no. 7 is close to zero. The reduced values of M resulting from such structural schemes are due to the fact that the structure is statically determinate for any possible progressive damage. There is not the possibility to have higher values of M in such configurations, as well as the cross-section areas of another rod tending to be zero. Such scenarios are represented by the points of Figure 4(c) that are in a transition area between the limit case C and the tail of limit case A (NSCI up to 0.89 and M ranging between 1.5 and 3).

The box plot of Figure 5(b), relating to limit case D, i.e., the one with large complexity and the lowest values of M, clearly shows that a differentiation of the axial stiffnesses across the various rods is present. Top and bottom chords, i.e., rods no. 1 and 2 and 4 and 5, have smaller cross-section areas with respect to the other structural components. Even if large cross-section areas result from the outlier data related to the previous rods (up to 18 times the cross-section area of rod no. 5), the median value is around (or smaller than) one. Since the bounds of each notch are very close to each other, the previous assertion is true with high level of confidence. Small size chords and stiff diagonal rods favor the redistribution of the force through the upper and lower part of the truss both in undamaged and in damaged scenarios. In the case of damage on diagonal elements, there is still the way of redistributing the loads on the chords. In the classical design of a cantilever truss, the chords are the elements into which compression and tension force equilibrates the couple induced by the load on the top of the cantilever: they appear to be the most important elements of the truss. In case of damage, such loads must be transferred on the diagonal elements that, usually, are not stiff enough because they are longer and oblique; this implies large displacements as long as damage increases. In the limit case herein highlighted, large axial stiffness is attributed to diagonal elements in such a ways to redistribute properly the loads in case of removal of a rod. Observing the curves of Figure 6(a), the largest effects are due to the removal of such chords that have smaller cross-section area. For example, in case of damage on rod no. 1, which is the top chord closer to the external pin, the traction forces in the upper chord (rods no. 2

and 3) transit through the oblique rod no. 10, increasing, meanwhile, the compression force on rod no. 7 which is equilibrated through tension on diagonal rod no. 11. The previous description serves to highlight the fact that, under such damage, the overall length of the path made by the force increases and, consequently, the stiffness tends to decrease, causing larger displacements.

As reported in the previous sections, the evaluation of the increment of deformation work can be intended as a measure of the robustness of the system. The average response of the system, i.e., the effect of a random damage or removal, is taken into consideration in the parameter M. Following the results of the analysis, which shows that an increment of the efficacy of all the load paths, i.e., an optimization in the distribution of the loads within the statically indeterminate truss, reduces the overall impact of a random damage. In addition, since the loads paths are more effective, the importance of each rod turns out to be accurately weighted insomuch that a progressive damage on one element produces a quasi-linear increment of the deformation work. Anyway, the results found in the analysis, i.e., stiff diagonal elements and relatively small chords, may appear to be in contrast with the current practice. This culture of disagreement is not new in discussions concerning about structural robustness (Masoero et al., 2010).

6 Conclusions

In summary, the study of the response of a truss structure to damage has been performed. The elastic energy was used as a measure for assessing the effects of progressive damage. In order to better evaluate the overall response, the average measure of the increment of deformation work was considered through the parameter M. In addition, the relationship between the value of the Normalized Structural Complexity Index and M was investigated. The former parameter is a measure of the efficacy of the load paths across the structure. A truss cantilever made of eight nodes and 15 rods was used throughout the calculations,

It has been found that, despite the linearity of the material and of the damaging process, the overall behavior of a truss under damage is nonlinear. This behavior is typical of statically indeterminate structures. For example, Biondini et al. (2008b) illustrated the equilibrium paths of damaged truss structures: depending on the position of the damage, nonlinear phenomena may be observed.

Anyway, despite the relative simplicity, the use of linear elastic relationship in the evaluation of the response of a construction is useful in structural engineering for making design choices. This approach is suited for screening all the possible structural solutions before setting a complete numerical model onto which more accurate (but costly) non-linear analyses can be performed. The use of linear elastic analyses is suggested by design codes, for example, for earthquake resistant buildings. The response of a construction to a seism is far from being linear; anyway, the codes allow the use of linear elastic analysis provided that the seismic spectrum is corrected by a behavior factor, which depends on the capability of the structure in dissipating energy. This permits to avoid explicit inelastic structural analysis and to simplify the preliminary design. Powell (2009) suggested that linear analysis is sufficient to assess the possibility that the sudden removal of a column from a structure turns into a disproportionate collapse.

Obviously, the structural response after damage depends on how the truss scheme works. As much as there is uniformity in the load paths, which are multiple in a statically indeterminate structure, the importance of each rod decreases. Because of that, the parametric analysis focusing on the relationship between damage and structural complexity gives evidence of the fact that, as much as the Normalized Structural Complexity Index increases, the behavior of the randomly damaged scheme improves. Smaller displacements and quasi-linearity in the response are recorded.

The idea of increasing the robustness of the structure by incrementing the degree of redundancy has been analyzed in detail in the past and is still debated. Early researches illustrated the positive effects of redundancy on systems, but indicated the degree of redundancy as a bad indicator for robustness (Frangopol and Curley, 1987). Recently, the topic has been analyzed in probabilistic terms, highlighting the fact that the redundancy has to be associated to ductility and over-strength to turn into robustness (Bertero and Bertero, 1999). Schafer and Bajpai (2005) showed that the role of redundancy in an intact structure is limited and an increase of redundancy may be associated to an increase in epistemic uncertainty, which reduces the vulnerability but not necessarily increases the resilience (Elms, 2004).

The implementation of such results in the construction engineering would permit the building of structures that are more robust against random damages Biondini and Frangopol (2014); Ghosn et al.

(2010); Zhu and Frangopol (2012). The ability of the structural engineer, in this sense, consists of its capacity to combine materials (concrete, reinforcement bars, tendons, steel sections,...), cross-section sizes, and structural details (confinement of nodes and reinforcement bar position in concrete trusses, steel connections) in order to respect the distribution of stiffnesses found in the complexity analysis.

At present, the robustness can be assessed and the effects of element removal can be precisely evaluated through fully probabilistic structural calculations. In the viewpoint of progressive damage, complex structures have the quality to actively respond to the damage phenomenon at its initial stages, differently from what is seen in the majority of the statically indeterminate structures. In the future, the possibility to implement the approach herein proposed in preliminary design stages has to be considered. Applications to more complex real structures, such as buildings and bridges, are necessary in order to show the practical applicability and usefulness of theoretical concepts and methods herein presented. A design procedure able both to tackle random damages and to optimize the structure is a sustainable strategy that would prevent global failures in structures and infrastructures. In addition, efforts shall be spent in determining a ductility parameter, similar to the behavior factor in seismic engineering, able to account for the redistribution of strengths within the structure as much as the damage progresses. This would allow a relatively simple method for designing robust structures.

Aknowledgements

This work has been funded under the framework of the "PRIN 2011. Built Heritage Information Modelling/Management - BHIMM". The author gratefully thank the three anonymous reviewers and the editor, Prof. D. Hills, for their suggestions and observations on the manuscript.

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